A Laser System to Measure Rocket Attitude in Flight

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A new concept, employing ground-based lasers, for attitude measurement of flight vehicles on test ranges is described. Two ground-based laser stations are required. Each station consists of a standard laser radar unit with the addition of a continuous-wave laser as well as transmitting and detecting optics for the cw laser. Cooperative vehicles incorporating two roof prisms, in addition to standard retroreflecting elements for tracking, are required. The technique is directly applicable to vehicles exhibiting roll rates on the order of 20 rev/s or higher. Computer simulation of the system, combined with a six-degree-of-freedom simulation of a tactical rocket, indicates that angular accuracy on the order of 0.1 deg may be attained. These simulations did not include considerations of radiometry, which would degrade the results somewhat. Laboratory and field experiments, both involving rocket models which were fixed in space, yielded an angular accuracy slightly better than 0.1 deg.

Nomenclature

B	= axial distance along vehicle
C	= distance in a plane perpendicular to vehicle axis
P	= represents the set X_m, Y_m, Z_m, δ_1, and δ_2
R	= radius of vehicle
t	= time
X, Y, Z	= coordinates in Earth-fixed system
β	= angular separation between two prisms
γ	= skew angle of prism
δ_1, δ_2	= geometric pitch and yaw, respectively
Δ	= increment, generally of time Δt
η, ξ, ω	= coordinates in vehicle-centered coordinate system
θ	= incremental roll angle
λ	= area ratio
μ	= angle of incidence
σ	= angle between laser beam and vehicle roll axis, aspect angle
ϕ	= represents the set $X_m, Y_m, Z_m, \delta_1, \delta_2$, and Ω
Ω	= roll rate

Subscripts

i	= denotes a particular ground station
j	= denotes the other ground station
K	= indicates range of variables
m	= indicates vehicle

Introduction

FLIGHT test programs of some proposed weapons systems will require accurate attitude measurements throughout a significant portion of the trajectory. The current techniques for attitude measurement include the use of fixed cameras and a variety of onboard instrumentation. Although fixed cameras provide quite accurate attitude measurements, their use is generally restricted to a relatively short portion of the overall trajectory.[1,2] Cinetheodolites can be used to cover essentially all of the trajectory; however the accuracy of attitude inferred is somewhat less than obtained with fixed cameras. Both of these techniques require photointerpretive data reduction, which is primarily manual, leading to relatively high costs and slow turnaround. Onboard instrumentation requires a telemetry system or recorder which must generally be considered expendable.[3] This paper describes a laser technique for automated measurement of flight vehicle position and attitude which offers the potential of improved accuracy for attitude measurements in real time with a minimum of expendable hardware.

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System Concept

Two ground-based laser tracking stations are required. Typically these stations incorporate a pulsed laser, transmitter, and detector optics all situated on an elevation over azimuth mount. The return signal from the vehicle must generally be enhanced through the use of conventional corner cubes, reflective tape, or paints located on the vehicle. Such devices are current technology, typified by the Precision Automated Tracking System (PATS).[4] In the following presentation it is assumed that the tracker will provide the space position of the vehicle. In order to determine attitude, each of the ground stations is additionally equipped with a continuous wave laser; and two roof prisms are located onboard the test vehicle. For vehicles exhibiting roll rates equal to or greater than the desired attitude data rates, the prisms are simply inlet into the surface of the round at a convenient location (e.g., dummy warhead). In the development which follows, we will assume that this is the case.

Consider the 90-deg roof prism shown in Fig. 1. For this application, the two surfaces which are shown crosshatched are silvered. Two lines emanating from the center of each silvered surface and perpendicular to their respective surface define a plane, termed here the plane of the retroreflector. A simple ray trace shows that a ray of light incident on one of the silvered surfaces and describing a path parallel to the plane of the retroreflector is generally reflected off the second surface back to the origin. If a rolling vehicle is equipped with a 90-deg roof prism and tracked so that it is continuously illuminated by a cw laser, it is clear that, a return signal of the cw radiation will be returned to the laser site each time the plane of the retroreflector passes through this ground station. The signal will be in the form of a pulse, the width of which depends on the beam divergence angle, the range, the optical quality of the roof prism, and the roll rate of the vehicle. The frequency of the pulses is directly related to the roll rate.

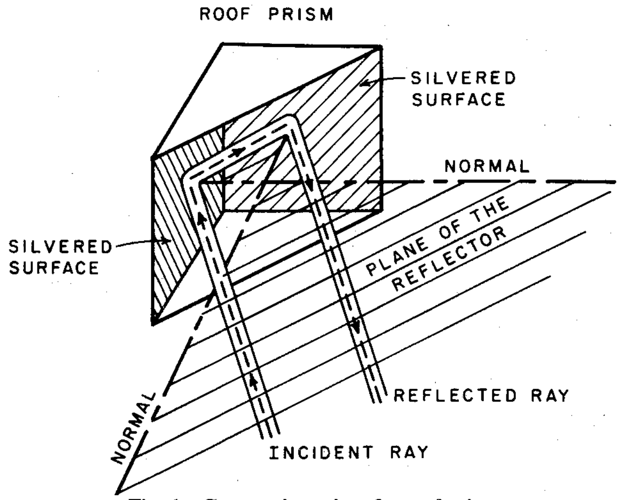

Fig. 1 Geometric optics of a roof prism.

A test range equipped with two tracking stations is illustrated in Fig. 2. A relationship between the return pulses received at the two stations will now be derived for a vehicle fixed in space, rotating at a constant angular rate, and equipped with a single roof prism oriented such that the plane of the reflector contains the roll axis of the vehicle. Two coordinate systems will be utilized as shown in Fig. 2. The Earth-fixed system is defined with the origin located at the launch point; positive Z axis in the vertical upward direction; positive Y axis in the downrange direction; and the X axis in the crossrange direction to provide a right-handed system. A vehicle-centered system is defined with the origin located at the intercept of a plane bisecting the 90-deg prism and the vehicle's roll axis. The ω axis coincides with the roll axis of the vehicle, positive toward the nose; η perpendicular to ω and parallel to the XY plane, positive toward the positive X direction; and ξ perpendicular to ω and η to form a right-hand orthogonal system. The components of the position vectors of the ith ground station and the vehicle in the Earth-fixed system are (X_i, Y_i, Z_i) and (X_m, Y_m, Z_m), respectively. Using the transformation matrix between the two systems, the components of the ith ground station in the vehicle-centered system are

$$\eta_i = \cos(\delta_2)(X_i - X_m) - \sin(\delta_2)(Y_i - Y_m) \tag{1a}$$

$$\omega_i = \sin(\delta_2)\cos(\delta_l)(X_i - X_m) + \cos(\delta_2)\cos(\delta_l)(Y_i - Y_m) + \sin(\delta_l)(Z_i - Z_m) \tag{1b}$$

$$\xi_i = -\sin(\delta_2)\sin(\delta_l)(X_i - X_m) - \cos(\delta_2)\sin(\delta_l)(Y_i - Y_m) + \cos(\delta_l)(Z_i - Z_m) \tag{1c}$$

where δ_l and δ_2 represent the geometric pitch and yaw, respectively, defined as in Fig. 2a. Referring to Fig. 2b, it may be seen that the time interval between pulses returned to the i and j ground station is

$$\Delta t_{ij} = \frac{1}{\Omega}\left[\arctan\left(\frac{\xi_i}{\eta_i}\right) - \arctan\left(\frac{\xi_j}{\eta_j}\right)\right] \tag{2}$$

Note that substitution for the ξ_i and η_i using Eq. (1) yields an equation in terms of relative position of the stations and the vehicle, and the geometric pitch and yaw. A third ground station could be used to provide a similar relationship yielding two equations and two unknowns (δ_l, δ_2) where it is assumed that the relative positions are obtained from the laser tracker, and the roll rate is inferred from pulses returned to a single station. Unfortunately, simultaneous solution of these two equations is relatively insensitive to yaw variation for reasonable third station locations and suffers the disadvantage of requiring three ground stations.

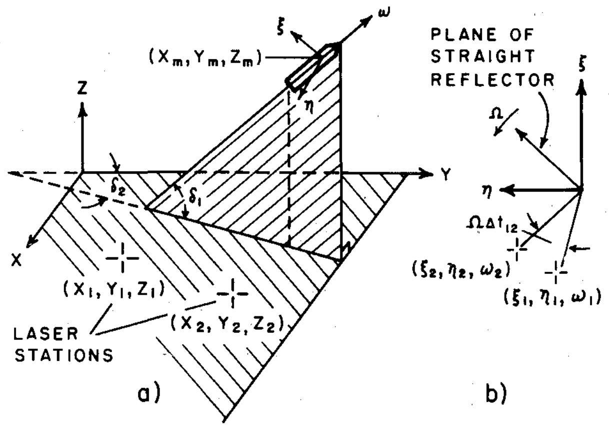

Fig. 2 Geometry for the development of the mathematical model.

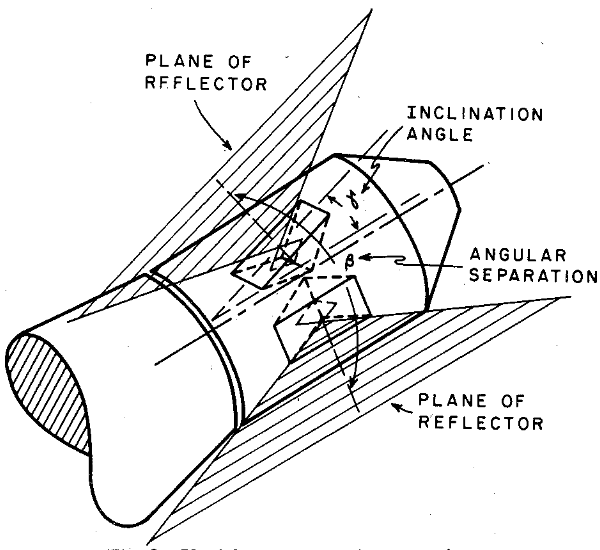

Fig. 3 Vehicle equipped with two prisms.

Consider now the addition of a second roof prism to the vehicle, as indicated in Fig. 3, with an angular separation of β relative to the first reflector and a skew of γ. The geometry of the prisms is shown in more detail in Fig. 4. Assume that at time t_l the plane of the straight retroreflector passes through the ith ground station. As the vehicle continues to roll the plane of the second reflector eventually passes through the same ground station at time t_2. It is apparent from the figure that the vehicle must roll a distance $\beta + \theta$ so that the time interval between the two pulse receptions is $\Delta t_{ii} = (1/\Omega)(\beta + \theta_i)$.

Figure 4 also indicates that $\tan\sigma_i = R/B_i$ and $\tan\gamma = C_i/B_i$. Therefore, $\theta_i = \arcsin(\tan\gamma)/(\tan\sigma_i)$. By noting that

$$\tan\sigma_i = (\eta_i^2 + \xi_i^2)^{1/2}/\omega_i$$

one obtains the result

$$\Delta t_{ii} = \frac{1}{\Omega}\left[\beta + \arcsin\left(\frac{\omega_i \tan\gamma}{(\eta_i^2 + \xi_i^2)^{1/2}}\right)\right] \tag{3}$$

which can be expressed in terms of δ_l and δ_2 using Eqs. (1).

This equation is very similar to the Yawsonde formulation because both approaches involve the relationship of planes, fixed with respect to the vehicle, relative to a remote point.[3] The Yawsonde approach differs from the concept presented here in as much as it involves onboard detectors and

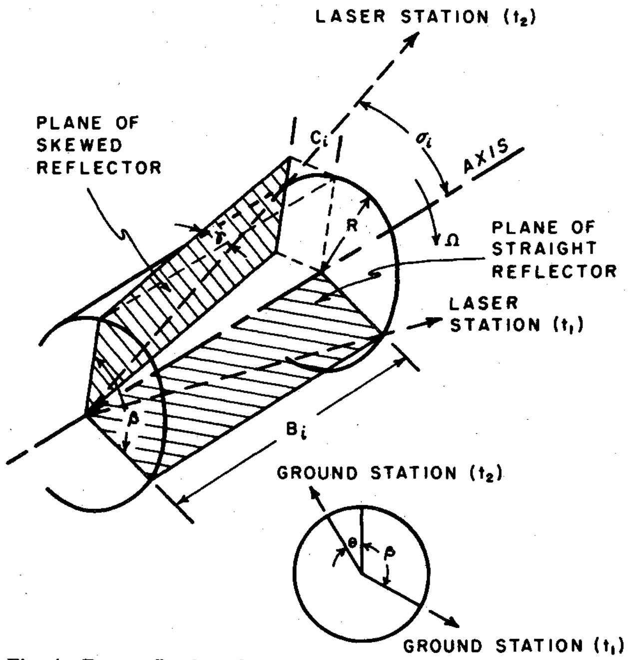

Fig. 4 Retroreflection planes referenced to a laser ground station at two different times.

telemetry. Equations (2) and (3) may be solved numerically for δ_1 and δ_2 assuming once again that Ω, $X_i - X_m$, $Y_i - Y_m$, $Z_i - Z_m$, Δt_{12} are known. It should be noted that Eq. (3) applies for both ground stations, thus providing two equations in two unknowns. These two equations can be solved for geometric pitch and yaw.[5,6] Preliminary studies indicate that for some geometric relationships between the ground stations and the vehicle, and for some vehicle attitudes, this is the preferable approach. On the other hand, for shallow trajectories and small pitch angles, it appears that simultaneous solution of Eq. (2) and one equation of the form of Eq. (3) is the more desirable approach.

The foregoing geometric relationships were developed using a static analysis which assumes a fixed vehicle position, constant geometric pitch and yaw, as well as a constant roll rate during the period required to measure the time intervals Δt_{ii} and Δt_{ij}. In order to estimate the influence that dynamic effects and system errors would have on attitudes inferred using the static analysis, two studies were conducted. The first involved a sensitivity analysis, whereas the second involved a computer simulation of the system incorporating a six-degree-of-freedom trajectory simulation for the vehicle.[7] For the sensitivity analysis, let Δt_{ii}^0 and Δt_{ij}^0 denote the actual time intervals between the pulses received from a dynamic vehicle. If Δt_{ii} and Δt_{ij} are the corresponding intervals for a static model with the vehicle position and attitude matching the values which existed at the beginning of the return pulse sequence, the following first-order expansions may be obtained

$$\Delta t_{ii}^0 \simeq \Delta t_{ii} + \sum_K \left(\frac{\partial \Delta t_{ii}}{\partial \phi_K}\right)_{t_1} (\phi_{K,t_1} - \phi_{K,t_2}) \tag{4}$$

and

$$\Delta t_{ij}^0 \simeq \Delta t_{ij} + \sum_K \left(\frac{\partial \Delta t_{ij}}{\partial \phi_K}\right)_{t_1} (\phi_{K,t_1} - \phi_{K,t_3}) \tag{5}$$

where t_1 is the observed time of reception of a pulse from the straight reflector at station i; t_2 is the observed time of reception of a pulse from the skewed reflector at station i; and t_3 is the time of reception of a pulse from the straight reflector at station j. ϕ_K is used to represent the variables δ_1, δ_2, X_m, Y_m, Z_m, and Ω of the dynamic vehicle. ϕ_{K,t_1} indicates that the variable is evaluated at time t_1 and terms of the form $(\partial \Delta t_{ii}/\partial \phi_K)t_1$ are the various sensitivities evaluated at t_1. Considering the Δt_{ij} sensitivities and letting P_K represent the ϕ_K variables with the exception of Ω, the sensitivities are obtained by differentiating Eq. (2) yielding

$$\frac{\partial \Delta t_{ij}}{\partial P_K} = \frac{1}{\Omega}\left[\frac{\left(\eta_j \dfrac{\partial \xi_j}{\partial P_K} - \xi_j \dfrac{\partial \eta_j}{\partial P_K}\right)}{(\eta_j^2 + \xi_j^2)} - \frac{\left(\eta_i \dfrac{\partial \xi_i}{\partial P_K} - \xi_i \dfrac{\partial \eta_i}{\partial P_K}\right)}{(\eta_i^2 + \xi_i^2)}\right] \tag{6}$$

Differentiation of Eq. (3) yields

$$\frac{\partial \Delta t_{ii}}{\partial P_K} = \frac{\tan\gamma\left[\dfrac{\omega_i}{\eta_i^2+\xi_i^2}\left(\eta_i \dfrac{\partial \eta_i}{\partial P_K} + \xi_i \dfrac{\partial \xi_i}{\partial P_K}\right) - \dfrac{\partial \omega_i}{\partial P_K}\right]}{\Omega(\eta_i^2 + \xi_i^2 - \omega_i^2 \tan^2\gamma)^{1/2}} \tag{7}$$

Sensitivities with respect to Ω, γ, and β take different forms. The various partial derivatives in Eqs. (6) and (7) may be expanded in terms of Eqs. (1).

An upper bound can be established for the difference between the observed and static time intervals:

$$|\Delta t_{ii}^0 - \Delta t_{ii}| \le \left[\sum_K \left(\frac{\partial \Delta t_{ii}}{\partial \phi_K}\right)^2_{t_1} (\phi_{K,t_1} - \phi_{K,t_2})^2\right]^{1/2} \tag{8}$$

and

$$|\Delta t_{ij}^0 - \Delta t_{ij}| \le \left[\sum_K \left(\frac{\partial \Delta t_{ii}}{\partial \phi_K}\right)^2_{t_1} (\phi_{K,t_1} - \phi_{K,t_3})^2\right]^{1/2} \tag{9}$$

An estimate of the maximum error introduced into the determination of attitude due to the use of the static model for dynamic situations can be made by solving Eqs. (2) and (3) using the range of time intervals given by Eqs. (4) and (5).

In order to proceed further, the following information must be provided: ground station positions, vehicle position, attitude, and roll rate, retroreflector geometry, and perturbation values. Rather than arbitrarily assuming some vehicle dynamics, a six-degree-of-freedom trajectory simulation of a research rocket (ARROW) was exercised. This program was modified to provide a simulation of the laser system in as much as each occurrence of retroreflection from a roof prism to a ground station was recorded. In addition, the values of the sensitivities $\partial \Delta t_{ii}/\partial \phi_K$ and $\partial \Delta t_{ij}/\partial \phi_K$ were computed at the time of occurrence of each return pulse. Pulse reception times and position data generated by the simulation were then used in Eqs. (2) and (3) to determine δ_1

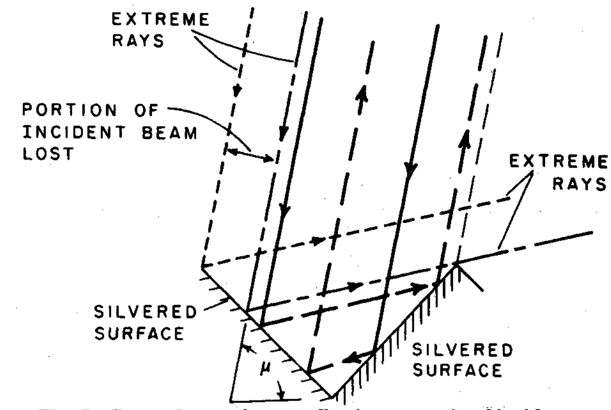

Fig. 5 Dependence of retroreflection on angle of incidence.

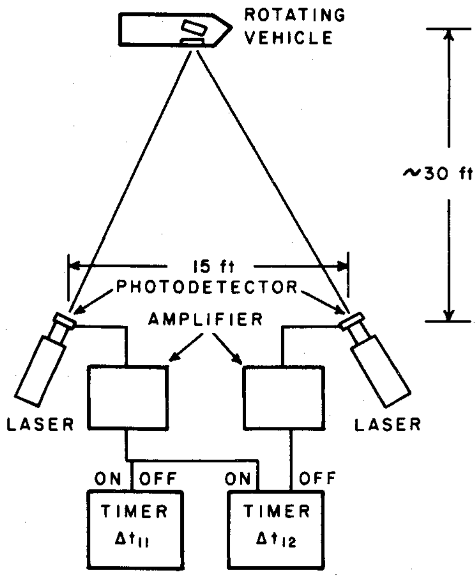

Fig. 6 Experimental arrangement.

and δ_2. These values were compared with the attitude data generated by the simulation.

The results of these particular studies are summarized in the following. In terms of the Earth-fixed coordinates shown in Fig. 2, the ground station locations used in the simulation were $X_l = 3000$ ft, $Y_l = 0$, $Z_l = 0$, $X_2 = 3000$ ft, $Y_2 = 2000$ ft, and $Z_2 = 0$. The reflector geometry incorporated a skew angle of 15 deg and an angular separation of 60 deg. Errors introduced in the attitude determination due to dynamic effects are dominated by roll rate variations, whereas errors introduced by retroreflector geometry uncertainties are predominantly related to the angular separation.

The use of the static data reduction technique introduced less than 0.1-deg error into the pitch and yaw determination in that portion of the trajectory where the roll rate exceeded 25 rev/s and the angular rates of pitch and yaw were less than 360 deg/s. Through most of the trajectory wherein the system could be utilized the error was nearly an order-of-magnitude less. It should be noted that no attempt was made to model the optical properties of the beam, atmosphere, or detector, all of which would lead directly to errors in time of pulse reception which would in turn lead to larger attitude errors.

The system has a geometric limitation based on the optics of the roof prisms. Figure 5 illustrates several ray paths through the roof prisms for rays that are not normal to the front face of the prism. It may be seen that for collimated incident radiation, a portion of the beam located between the two extreme rays is not returned to the source. As the incident ray deviates more from normal, the portion of the incident beam returned to the source decreases. The ratio of the area of the returned beam to the incident beam is $\lambda = 2/[1 + \tan(135 \text{ deg} - \mu)]$.

It is seen that the region up to ± 30 deg off normal is the useful working range, if λ is arbitrarily limited to the range $1 \leq \lambda \leq 0.5$. This range could be extended by adding more reflectors in the two retroreflection planes at different orientations, however there would be a rather severe alignment requirement.

Experimental Program

A series of experiments were conducted to verify the concept and in an attempt to identify unforseen problems. These experiments are described in the following.

Laboratory Experiment

Two static laser ground stations and a vehicle model were fabricated. In terms of Fig. 2, the ground stations were located at $X_l = 30$ ft, $Y_l = 0$, $Z_l = 0$, $X_2 = 30$ ft, $Y_2 = 15$ ft, and $Z_2 = 0$, and the rocket was systematically positioned at various X_m, Y_m, and Z_m locations. Each ground station incorporated a 3-mW helium neon laser, spatial filter, and collimator with a mounting ring for the photodetectors.

The vehicle model consisted of a short heavy-walled aluminum cylinder. Two holes separated circumferentially by 90 deg were bored through the wall of the cylinder at one axial position. One hole accommodated a mount for a skewed reflector which allowed the angle γ for this reflector to be adjusted to a particular value for various experiments. A steel drive shaft was attached to the base of the cylinder along its axis. The shaft was supported on two ball bearings which were, in turn, attached to a base plate. An electric gear motor was attached to the base plate and coupled to the drive shaft through a flexible coupling. The model was mounted on an L-bracket which was in turn attached to a rotary table. This arrangement provided a rather crude but rigid elevation over azimuth mount for the model. Both lasers were mounted on heavy-duty metal tripods to facilitate positioning and alignment.

The output signals of the photodetectors were amplified and used to initiate and terminate counting on two digital timers. A schematic representation of the detection/timing circuit is shown in Fig. 6. Roll rate was measured using a signal from one laser ground station to start and stop one counter with every other retrurn pulse. The time interval between pulses received at two laser stations as the plane of the straight reflector passes through them was measured by starting the counter when a pulse was received at station 1, and stopping the counter when a pulse was received at station 2. Simultaneously, the time interval between pulse receptions from the straight and skewed reflectors at station 1 was measured.

Setting the angle γ and measuring the various angles to a high degree of accuracy ultimately required the use of lasers and large optical levers. The entire system including ground stations, model, and retroreflectors was eventually aligned to an angular accuracy of 0.05 deg. Experiments were conducted with the model located at two different positions downrange and pitch and yaw attitudes varying from -20 to $+20$ deg. The measured time intervals were used to determine the pitch and yaw attitude. These tests indicate that the pitch and yaw of a static system can be determined to essentially the same accuracy as the experimental error associated with aligning the various components.

Field Experiment

Fabrication of a full-scale prototype laser ground station has recently been completed. A cw neodymium-YAG laser and detector are incorporated on a standard laser tracker to form an attitude subsystem. A series of static tests of attitude measurement was conducted using the laser and detector developed for the prototype. The experiments conducted were similar to the laboratory tests. A replacement fuse for a 2.75-in. rocket was equipped with two roof prisms. One of the prisms was located such that its plane of retroreflection was parallel to and contained the axis of symmetry of the replacement fuse. The second prism was rotated (β) 10 deg from the first prism and skewed (γ) 15 deg. A ¼-in. drill rod shaft was located on the axis of the fuse and driven with a variable speed electric motor. The fuse shaft was mounted on bearings which were attached to a table that was, in turn, mounted on an elevation over azimuth mount. The replacement fuse was illuminated by the cw laser which was located 750 ft downrange and operated at power levels estimated to be between 10 and 20 W.

The experiment was conducted using a conventional rifle scope mounted on a special fixture to orient the axis of the fuse at 90 deg to the laser beam. A theodolite mounted on the target table was then zeroed and used thereafter to set the yaw attitude of the fuse axis with respect to the laser beam. Two counters were used to measure both the interval between consecutive pulses and the interval between every other pulse returned to the detector. A simple flip-flop circuit was used to blank every other pulse to one counter providing a direct readout of the reciprocal of the roll rate. The other counter displayed the time interval between pulses which was used to determine yaw attitude. Since only one laser/detector station was used, only one aspect angle was determined. In almost all runs, the pitch was fixed at zero and the yaw was systematically varied up to ±35 deg off normal.

Data reduction involved using the equation.

$$\sigma = \arctan\left[\frac{\tan\gamma}{\sin(\Omega\Delta t - \beta)}\right] \tag{10}$$

which defines the aspect angle as a function of the two geometric constants, the roll rate and the measured time interval. The mean deviation between aspect angles determined using Eq. (10) and theodolite values was calculated for 65 measurements wherein σ was varied over a range from 55 to 125 deg. The value of the mean deviation obtained was 0.07 deg.

Summary

A concept for remote measurement of the attitude of rolling vehicles using two ground-based laser tracking stations and two roof prisms mounted onboard the vehicle has been investigated using both computer simulation and experimental techniques. The results indicate that, for vehicles exhibiting relatively high roll rates, geometric pitch and yaw may be determined to accuracies on the order of 0.1 deg. It appears that this system would be suitable for tests of artillery shells and some rockets.

The geometric relationship between the roof prisms onboard the vehicle as well as the relationship between the ground station sites and the anticipated trajectory can be optimized for particular test program requirements. Efficient retroreflection from roof prisms is limited to angles approximately ±30 deg off the normal to the front face of the prism which may limit the portion of the trajectory which can be covered by a single pair of ground stations. No specific range limitation has been identified; however it is apparent that there will be a limitation based on the radiometry which will be a function of: the laser power and collimation; the entrance aperature, working angle, and quality of the prisms; detector optics and efficiency; and atmospheric characteristics.

Acknowledgments

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References

[1] Campbell, V., "Evaluation of the Accuracy of Data Obtained from Fixed Cameras," U.S. Army Missile Command, RT-TR-69-17, July 1969.

[2] Sweigert, D. J. and Carmack, P., "Optical Lever Flight Instrumentation," AMC Rept., RT-TI-70-28, 1970.

[3] Mermagen, W. H., "Measurements of the Dynamical Behavior of Projectiles Over Long Flight Paths," *Journal of Spacecraft and Rockets*, Vol. 8, April 1971, pp. 380-385.

[4] Steele, W., "Multiple Laser Tracker Operations," Proceedings of the 9th Annual Electro-Optics/Laser Conference, 1977.

[5] Grubin, C., "Simple Algorithm for Intersecting Two Conical Surfaces," *Journal of Spacecraft and Rockets*, Vol. 14, April 1977, pp. 251-252.

[6] Grubin, C., "Satellite Attitude Determination by Simultaneous Line-of-Sight Sightings," *Journal of Spacecraft and Rockets*, Vol. 14, Oct. 1977, p. 640.

[7] Lain, D., "Rocket Attitude Determination Using Two Ground-Based Laser Tracking Stations," M.S. Thesis, University of Wyoming, 1975.